SLIDING MODE CONTROLLER DESIGN FOR KINETIC ENERGY KILL VEHICLE IN A HOSTILE ENVIRONMENT

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Abstract

Current guidance and control systems for interceptor missiles use algorithms that require knowledge of the target. These systems are not robust to target maneuvers and disturbances. In this paper, we investigate the use of a novel controller named a Sliding Mode Controller (SMC). We consider two types of interceptors or kinetic energy kill vehicles (KEKVs): a ballistic missile and a cruise missile. Sliding mode controllers are robust, simple guidance and control algorithms which can be applied to interceptors. After the interceptor flight path angle captures the line-of-sight angle, the SMC will follow the line-of-site angle regardless of external disturbances, plant uncertainties or measurement noise until the target missile is intercepted. The developed guidance and control algorithms require estimating or measuring only the interceptor missile flight path, line-of-sight angles and their derivatives. Estimation of the target position, velocity and acceleration is not required. We have validated the effectiveness of the designed sliding mode controllers through numerical examples.

Introduction

Kinetic energy kill vehicles are employed as interceptors in theater missile defense (TMD)[1]. A KEKV destroys an incoming ballistic missile as they impact. The interceptor's guidance and control system must provide autonomous target tracking and homing maneuvers to achieve a very accurate intercept. A two-dimensional missile-target engagement geometry[2] is shown in fig. 1, where $X0Z$ is the inertial frame, $\mathbf{V}_M$ and $\mathbf{V}_T$ are velocity magnitudes of the missile and the target (m/s), CG_M and CG_T are centers of gravity of the missile and the target, λ is the line-of-sight angle (*rad*), γ and β are flight path angles of the missile and the target respectively (*rad*), R_{TM} is a length of the line-of-sight or range (*m*), α is angle of attack (rad), n_c is the acceleration missile command (m/s^2), n_T is the target acceleration (m/s^2), u_d and u_a are divert and attitude control forces respectively.

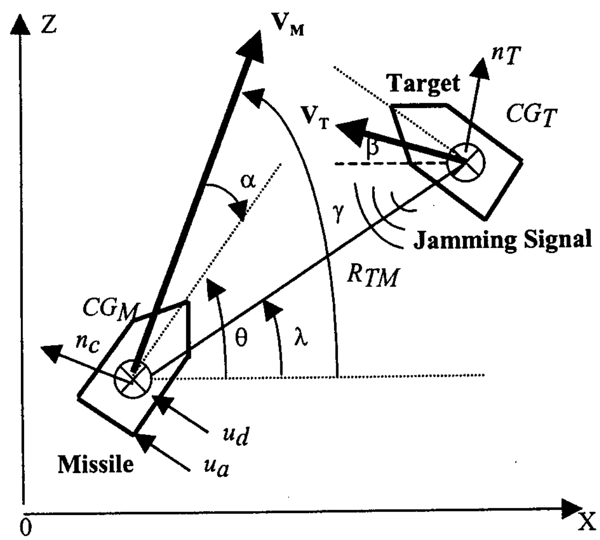

Fig. 1 Two-dimensional missile-target engagement geometry

The jamming signal is a random noise generated by the target. Traditionally, the acceleration missile command n_c is calculated as proportional or augmented proportional[2] navigation laws, advanced guidance law[2,3], linear quadratic optimal guidance law[4], etc. The acceleration missile command n_c is then executed by a flight control system to minimize the miss distance (the distance between the target and interceptor).

The simplest proportional navigation law[2] can be expressed in a format

$$n_c = NV_{CL}\dot{\lambda} \; , \tag{1}$$

where $N \in [3,5]$ is the navigation ratio, and V_{CL} is a magnitude of the missile-target closing velocity (m/s). It is well known[2] that implementing the proportional

navigation law, Eq. (1), requires the seeker to estimate $\dot{\lambda}$ as well as V_T, β, n_T. To estimate the values of the target parameters, high order Kalman filters[2,3] are usually designed. These filters should also address nonlinearities of the missile-target kinematics and dynamics. A maneuvering target (for instance, submunition) complicates predicting target motion. Sometimes these estimations are done by postulating the sinusoidal target weave motion[5]. A hostile environment affects the interceptor's motion. In particular, unknown disturbances (for instance, wind torque during atmospheric flight) and parametric uncertainties including control system failures (for instance, damage of the aerodynamic surfaces) affect the interceptor's motion and tracking dynamics. A jamming signal (noise) which is generated by the target can create difficulties for the seeker in estimating the values of $\dot{\lambda}$, V_T, β and n_T. As a result, the guidance and control system of the KEKV becomes cumbersome and extremely sensitive to the accuracy of parameter estimation for target motion and unknown disturbances.

One of the promising nonlinear control algorithms that is robust to disturbances and plant uncertainties is Sliding Mode Control[6-8]. The purpose of a Sliding Mode Controller (SMC) is to drive a nonlinear plant's trajectory onto a prescribed (user-chosen) surface, named the sliding surface, in the state space and to maintain the plant's state trajectory on this surface thereafter. The motion of the system on the sliding surface is called the sliding mode. The equation for the sliding surface must be selected such that the system will exhibit the desired (given) behavior in the sliding mode independent of unwanted parameters (plant uncertainties and disturbances). Usually the SMC is characterized by a high frequency switching control function that provides the system's motion in the sliding surface[6-8]. The discontinuous nature of a SMC is suitable KEKV control systems that use divert and attitude thrusters. A SMC can be implemented in a pulse-width modulation (PWM) format to enforce a certain frequency of thruster firing. However, guiding the KEKV by aerodynamic surfaces requires continuous implementation of a discontinuous SMC[9,10]. SMCs have been successfully applied to address significant plant uncertainties and disturbances for various space and flight control problems[11,12].

SMC Design for KEKV

Robust, simple guidance and control algorithms for kinetic energy kill vehicles operating in hostile environments can be developed in sliding modes. The simplicity of the SMC's design is based on the fact that estimating the target's parameters is not required. Only λ, γ, $\dot{\lambda}$ and $\dot{\gamma}$ must be measured or estimated for the interceptor. The robustness of the SMCs has been demonstrated on system operation in sliding mode[6-11]. In designing the SMC for a KEKV, two goals are proposed:

1. Direct the velocity vector of interceptor, $\mathbf{V}_M$, to the target by following the line-of-sight angle λ to the interceptor flight path angle γ regardless of disturbances and uncertainties,
2. Reduce drag force by directing the interceptor axis of symmetry to the center of gravity of the target regardless of disturbances and uncertainties.

We have evaluated SMC designs for two types of KEKVs, a ballistic missile and a cruise missile type.

SMC Design for a Ballistic Type KEKV

A simplified mathematical model is obtained for the ballistic type interceptor in the pitch plane.

Mathematical Model of a Plane Motion

A simplified translational motion of the ballistic type interceptor in the pitch plane (fig. 1) is described as follows:

$$\begin{cases} \dot{v}_x = -\dfrac{1}{m}(u_a + u_d)\sin\theta + \dfrac{\psi_x}{m}, \\ \dot{v}_z = \dfrac{1}{m}(u_a + u_d)\cos\theta - g + \dfrac{\psi_z}{m}, \end{cases} \quad (2)$$

where

$\mathbf{V}_M = \{v_x, v_z\}^T$, m is the mass of the interceptor (*kg*), $\Psi = \{\psi_x, \psi_z\}^T$ is the vector of disturbance and aerodynamic forces (*N*), and $g = 9.81\, m/s^2$.

A simplified rotational motion of the ballistic type interceptor in the pitch plane (fig. 1) is described as follows:

$$\begin{cases} \dot{\theta} = q, \\ \dot{q} = -\dfrac{r}{J}u_a + \dfrac{\psi_\omega}{J}, \end{cases} \quad (3)$$

where

q the a pitch rate (*rad/s*), J is the moment of inertia of the interceptor ($kg \cdot m^2$), r is the distance between application of the attitude control force u_a and the interceptor center of gravity CG_M, and ψ_ω is the sum of aerodynamic and disturbance torque.

The output equations are

$$\begin{cases} y_1 = \gamma = \tan^{-1}\left(\dfrac{v_z}{v_x}\right), \\ y_2 = \theta. \end{cases} \tag{4}$$

Problem Formulation

To address the first goal for the SMC, we will provide the following tracking motion:

$$\lim_{t\to\infty}\left|y_{1d}(t) - y_1(t)\right| = \lim_{t\to\infty}\left|\lambda(t) - \gamma(t)\right| = 0, \tag{5}$$

where d denotes the desired output.

To address the second goal (attitude control problem), the SMC is designed to fit a desired tracking motion

$$\lim_{t\to\infty}\left|y_{2d}(t) - y_2(t)\right| = \lim_{t\to\infty}\left|\lambda(t) - \theta(t)\right| = 0. \tag{6}$$

We can observe that the rotational motion in not effected by divert control force u_d. Therefore, the second goal, Eq. (6), will be addressed first via the design of an attitude SMC, u_a. The first goal, Eq. (5), will be addressed next through the design of a divert SMC, u_d.

SMC Design

To address the attitude control problem, Eq. (6), the following sliding surface was designed

$$\sigma_2 = \dot{e}_2 + c_2 e_2 = 0, \tag{7}$$

where $e_2 = \lambda(t) - \theta(t)$.

The value of a parameter c_2 will be identified to provide the desired eigenvalue to a homogeneous linear time-invariant differential equation (7) that describes the tracking dynamics of a rotational motion on the sliding surface.

The control law u_a is designed to provide a finite-time convergence of the system's Eq. (3) trajectory to the origin in the $\sigma_2 -$ subspace. The system's Eq. (3) motion in the $\sigma_2 -$ subspace is derived as follows:

$$\dot{\sigma}_2 = \ddot{\lambda}(t) + c_2\dot{e}_2 - \frac{\psi_\omega}{J} + \frac{r}{J}u_a. \tag{8}$$

A candidate to the Laypuniv function is formed as follows:

$$Q_2 = \frac{\sigma_2^2}{2} > 0, \tag{9}$$

and its derivative is identified

$$\dot{Q}_2 = \sigma_2\dot{\sigma}_2 = \sigma_2\left(\ddot{\lambda}(t) + c_2\dot{e}_2 - \frac{\psi_\omega}{J} + \frac{r}{J}u_a\right). \tag{10}$$

Providing a finite-time convergence to $\sigma_2 = 0$ the control law u_a is designed to meet the following inequality[6-8]:

$$\dot{Q}_2 = \sigma_2\dot{\sigma}_2 \le -\hat{\rho}\left|\sigma_2\right|, \quad \hat{\rho} > 0 \tag{11}$$

Equivalent attitude control $u_{a_{eq}}$ is identified as follows[6-8]:

$$u_{a_{eq}} = \frac{J}{r}\left(-\ddot{\lambda}(t) - c_2\dot{e}_2 + \frac{\psi_\omega}{J}\right). \tag{12}$$

Meeting inequality (11) and assuming

$$\left|\hat{u}_{a_{eq}} - u_{a_{eq}}\right| \le N > 0, \tag{13}$$

the following SMC is designed:

$$u_a = \hat{u}_{a_{eq}} - \rho_2 sign\sigma_2, \tag{14}$$

where $\hat{u}_{a_{eq}}$ is $u_{a_{eq}}$ estimate, and $\rho_2 \ge N + \frac{J}{r}\hat{\rho}$.

Directing the interceptor velocity vector, $\mathbf{V}_M$, to the target, the divert SMC, u_d, is designed to meet Eq. (5). Differentiating y_1 we obtain

$$\dot{y}_1 = \Omega(.) + \frac{\cos\alpha}{m\left|\mathbf{V}_M\right|}u_d, \tag{15}$$

where

$$\Omega(.) = \frac{1}{\left|\mathbf{V}_M\right|}\left[\left(\frac{\psi_z}{m} - g\right)\cos\gamma - \frac{\psi_x}{m}\sin\gamma + \frac{\cos\alpha}{m}u_a\right],$$

$\left|\mathbf{V}_M\right| = \sqrt{v_x^2 + v_z^2}$ and $\alpha = \theta - \gamma$

The following assumptions are made

1. $\dfrac{\cos\alpha}{m\left|\mathbf{V}_M\right|} > 0$ in a reasonable flight domain,
2. corresponding internal (zero) dynamics are stable[13].

Then the relative degree is equal to one, and the sliding surface is designed as follows:

$$\sigma_1 = e_1 = 0, \tag{16}$$

where $e_1 = \lambda(t) - \gamma(t)$.

It is obvious that after the sliding surface in Eq. (16) has been reached, the conditions of Eq. (5) will be met. The control law u_d is designed to provide a finite-time convergence of the system's Eq. (2) trajectory to the origin in the $\sigma_1 -$ subspace. The system's Eq. (2) motion in the $\sigma_1 -$ subspace is derived as follows:

$$\dot{\sigma}_1 = \dot{\lambda}(t) - \frac{1}{\left|\mathbf{V}_M\right|}\left[\left(\frac{\psi_z}{m} - g\right)\cos\gamma - \frac{\psi_x}{m}\sin\gamma + \frac{\cos\alpha}{m}u_a\right] - \frac{\cos\alpha}{m\left|\mathbf{V}_M\right|}u_d. \tag{17}$$

A candidate to the Laypuniv function is formed as follows:

$$Q_1 = \frac{\sigma_1^2}{2} > 0, \tag{18}$$

and its derivative is identified

$$\dot{Q}_1 = \sigma_1\dot{\sigma}_1 =$$
$$\sigma_1\left\{-\frac{1}{|\mathbf{V}_M|}\left[\left(\frac{\psi_z}{m}-g\right)\cos\gamma-\frac{\psi_x}{m}\sin\gamma+\frac{\cos\alpha}{m}u_a\right]+\right.$$
$$\left.+\dot{\lambda}(t)-\frac{\cos\alpha}{m|\mathbf{V}_M|}u_d\right\}. \qquad (19)$$

Providing a finite-time convergence to $\sigma_1 = 0$, the control law u_d must be designed to meet following inequality[6-8]:

$$\dot{Q}_1 = \sigma_1\dot{\sigma}_1 \le -\tilde{\rho}|\sigma_1|, \quad \tilde{\rho} > 0 \qquad (20)$$

The equivalent divert control $u_{d_{eq}}$ is identified[6-8]as

$$u_{d_{eq}} = -u_a + \frac{m}{\cos\alpha}\left\{\left(-\frac{\psi_z}{m}+g\right)\cos\gamma+\frac{\psi_x}{m}\sin\gamma+\right.$$
$$\left.+|\mathbf{V}_M|\dot{\lambda}(t)\right\} \qquad (21)$$

Assuming

$$\left|\hat{u}_{d_{eq}} - u_{d_{eq}}\right| \le P > 0, \qquad (22)$$

the following SMC is designed to meet the inequality described in Eq. (20):

$$u_d = \hat{u}_{d_{eq}} + \rho_1 sign\sigma_1, \qquad (23)$$

where $\hat{u}_{a_{eq}}$ is $u_{a_{eq}}$ estimate, and $\rho_1 \ge P + \dfrac{m|\mathbf{V}_M|}{\cos\alpha}\tilde{\rho}$.

Simulation

The following numerical values represent a generic ballistic missile KEKV:

$$m = 70\,kg,\ J = 10\,kg\cdot m^2,\ r = 1\,m. \qquad (24)$$

The maximum values of attitude and divert thrust are:

$$U_{a\max} = 500\,N,\ U_{d\max} = 40000\,N. \qquad (25)$$

The following interceptor initial conditions were used during simulations:

$$\begin{cases} z(0) = 50{,}000m,\ x(0) = 10{,}000m,\ \theta(0) = -0.5\,rad, \\ \omega(0) = 0,\ V_x(0) = 2{,}000m/s,\ V_z(0) = -1{,}000m/s \end{cases} \qquad (26)$$

The initial position and velocity of a maneuvering target (submunition) were chosen as:

$$\begin{cases} z_T(0) = 40{,}000\,m,\ x_T(0) = 35{,}000\,m, \\ V_{xT}(0) = -750\,m/s,\ V_{zT}(0) = -100\,m/s \end{cases} \qquad (27)$$

The attitude SMC, u_a, was designed in accordance with Eqs. (7) and (14). This is

$$u_a = 500 sign\sigma_2,\ \sigma_2 = \dot{e}_2 + 2e_2 = 0. \qquad (28)$$

The divert SMC, u_d, was designed in accordance with Eqs. (16) and (23). This is

$$u_d = 40{,}000 sign\sigma_1,\ \sigma_1 = e_1 \qquad (29)$$

The results of the system "interceptor-target' simulation without disturbances and sensor noise are presented in figures 2 through 8. Very accurate kill and attitude alignment is demonstrated. The results of the system's simulation with disturbances, including jamming signals, and censoring noise are presented in figures 9 through 11. The line-of-sight angle, λ, is accurately followed by the interceptor flight path angle, γ, and the pitch angle, θ, regardless of the disturbances or measurement noise.

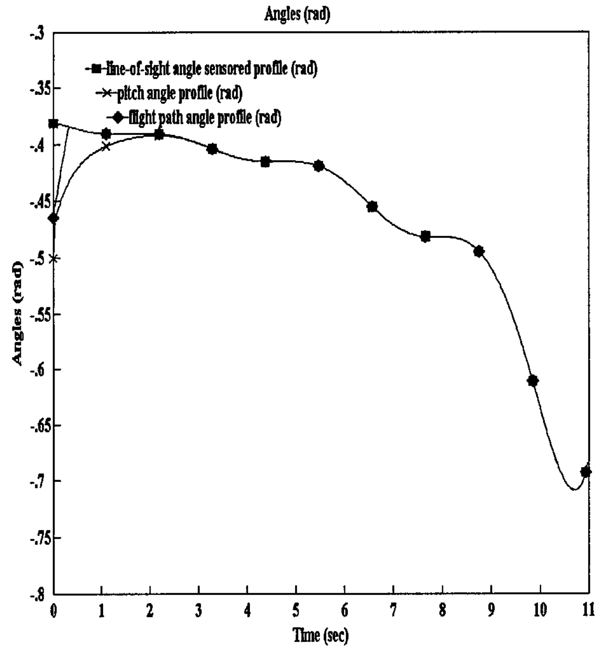

Fig. 2 Tracking of line-of-sight angle without disturbances and noise of measurement

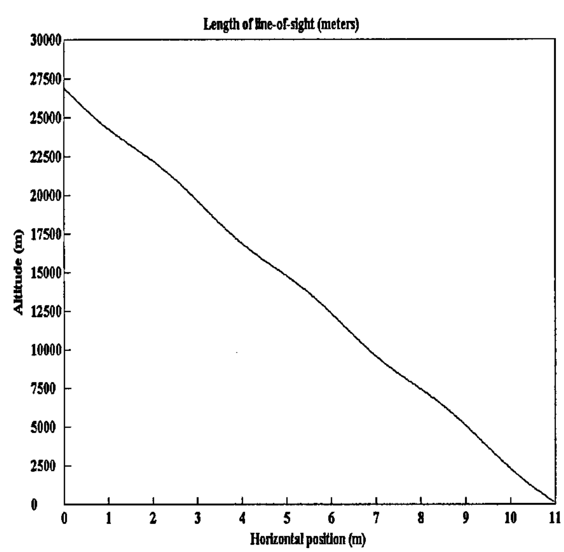

Fig. 3 Length of line of sight without disturbances and noise of measurement

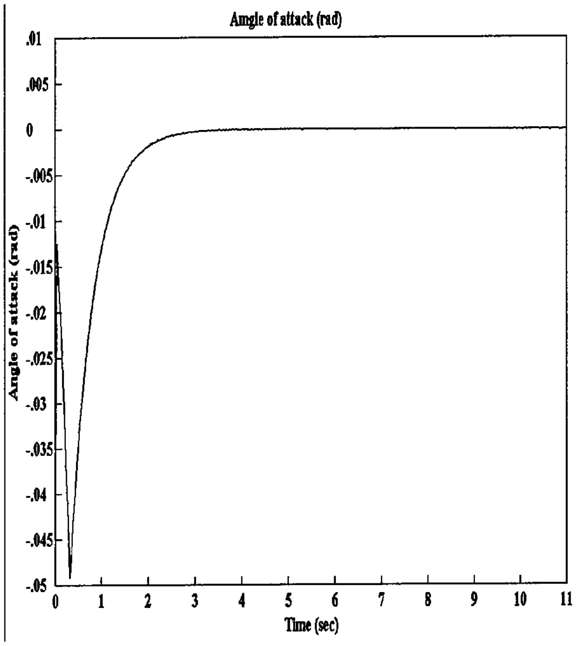

Fig. 4 Angle of attack without disturbances and noise of measurement

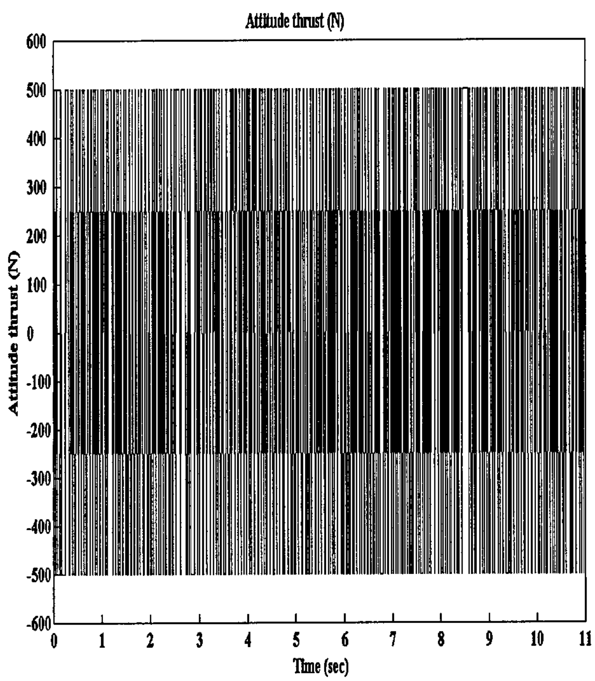

Fig. 5 Attitude thrust without disturbances and noise of measurement

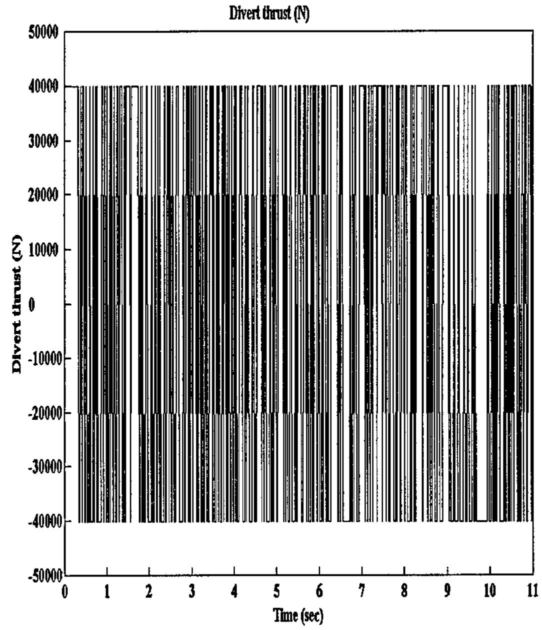

Fig. 6 Divert thrust without disturbances and noise of measurement

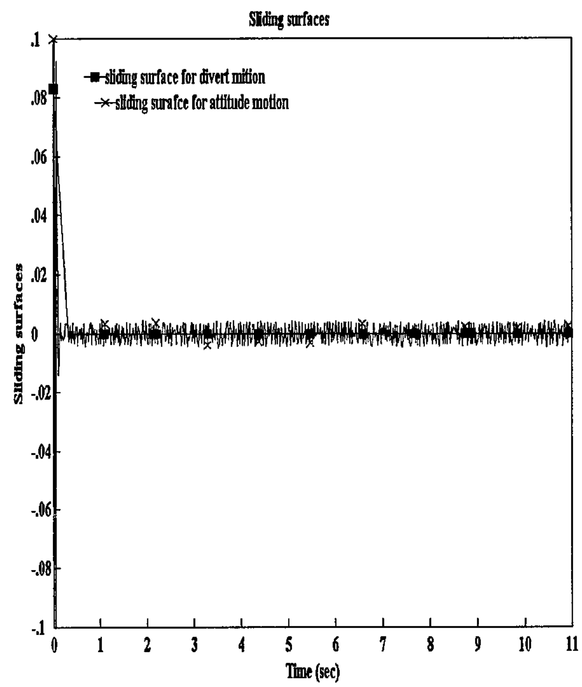

Fig. 7 Sliding surfaces without disturbances and noise of measurement

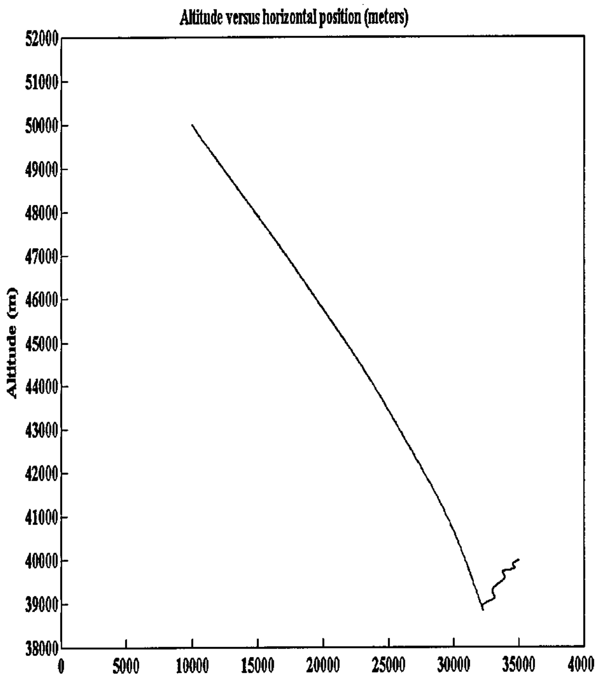

Fig. 8 Phase portrait "interceptor-target" without disturbances and noise of measurement

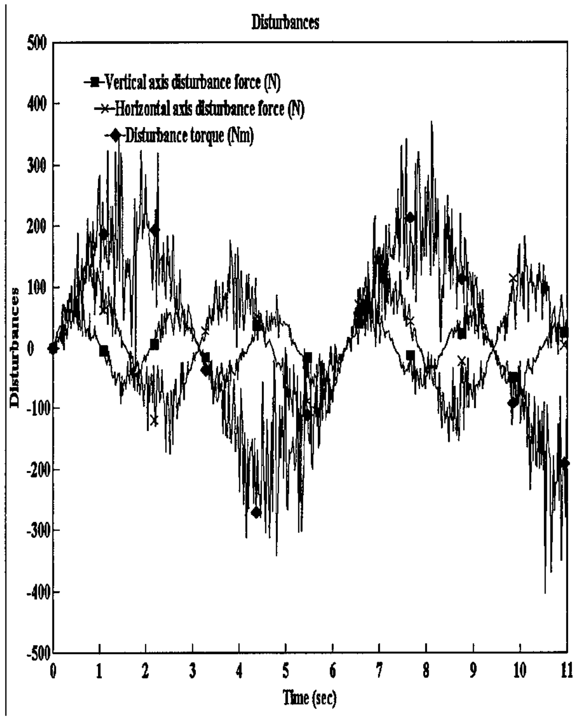

Fig. 10 Disturbances

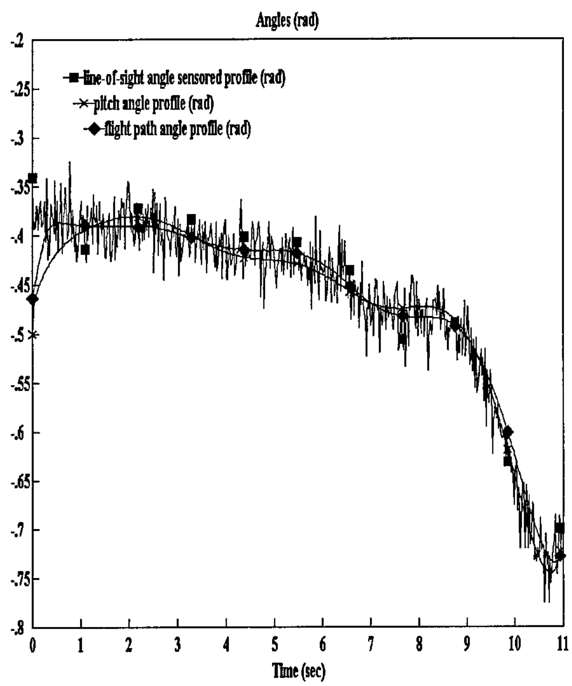

Fig. 9 Tracking of line-of-sight angle with disturbances and noise of measurement

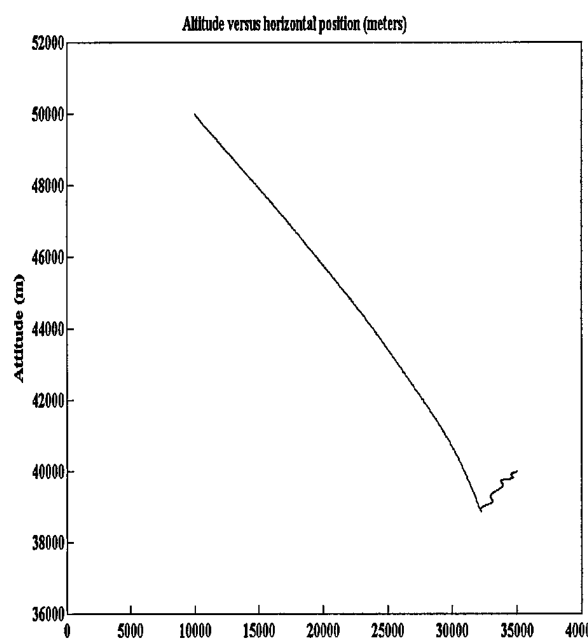

Fig. 11 Phase portrait "interceptor-target" with disturbances and noise of measurement

SMC Design for a Cruise Type KEKV

We now consider a strategic air defense missile interceptor (cruise type), using tailfin control.

Mathematical Model of a Plane Motion

The following simplified mathematical model represents the pitch-plane angular equations of motion of the cruise type interceptor[14].

$$\begin{bmatrix}\dot{\theta}\\ \dot{q}\\ \dot{\gamma}\end{bmatrix}=\begin{bmatrix}0 & 1 & 0\\ \frac{\bar{q}\cdot\bar{c}\cdot S}{J}C_{m\alpha} & \frac{\bar{q}\cdot\bar{c}^2\cdot S}{J|\mathbf{V}_M|}C_{mq} & -\frac{\bar{q}\cdot\bar{c}\cdot S}{J}C_{m\alpha}\\ \frac{\bar{q}\cdot S}{m|\mathbf{V}_M|}C_{z_\alpha} & 0 & -\frac{\bar{q}\cdot S}{m|\mathbf{V}_M|}C_{z_\alpha}\end{bmatrix}\begin{bmatrix}\theta\\ q\\ \gamma\end{bmatrix}+ \tag{30}$$

$$+\begin{bmatrix}0\\ 0\\ -\frac{\cos\gamma}{|\mathbf{V}_M|}g\end{bmatrix}+\begin{bmatrix}0\\ \frac{\bar{q}\cdot\bar{c}\cdot S}{J}C_{m\delta}\\ \frac{\bar{q}\cdot S}{m|\mathbf{V}_M|}C_{z_\delta}\end{bmatrix}\delta,\quad y_1=\begin{bmatrix}0 & 0 & 1\end{bmatrix}\begin{bmatrix}\theta\\ q\\ \gamma\end{bmatrix}$$

where S is the tailfin cross section area (m^2), $\bar{q}$ is the dynamic pressure (N/m^2), $\bar{c}$ is the missile diameter (m), $C_{m\alpha}$, C_{mq}, C_{mq}, C_{z_α}, C_{z_δ} are aerodynamic coefficients, m is the mass of the KEKV (kg).

Problem Formulation

Having only one control input: deflection δ of a tailfin, we can address only one goal: following the line-of-sight angle, λ, by the interceptor flight path angle γ. Addressing this goal we will provide the following tracking motion:

$$\lim_{t\to\infty}|y_{1d}(t)-y_1(t)|=\lim_{t\to\infty}|\lambda(t)-\gamma(t)|=0$$

by a corresponding design of a control input: deflection δ of a tailfin. This design will be accomplished in a continuous SMC format[9,15].

We can observe that the control input δ enters two equations of the system in Eq. (30). This fact can lead to a nonminimum phase case[13]. It is well known that traditional sliding mode control is not applicable to nonminimum phase systems[6,7]. Sliding mode control that is applicable to nonminimum phase systems is developed on the basis of dynamic sliding surface design[16].

Basics of Nonminimum Phase Output Tracking in Dynamic Sliding Surfaces[16]

The following nonlinear nonminimum phase plant is considered

$$\dot{x}=Ax+F(x,t)+bu,\quad y=Gx, \tag{31}$$

where $x\in\Re^n$ is the state vector, $u\in\Re^1$ is the control function, $y\in\Re^1$ is the controlled output, A,b,G are the constant matrices of corresponding dimensions, $\{A,b\}$ is a controllable pair, $F(x,t)\in\Re^n$ is a nonlinear time-dependent vector-function, so that $F(x,t)=F_1(x)+F_2(t)$, where $F_1(x)$: $\|F_1(x)\|\le N_1\|x\|$ is a matched nonlinear function and $F_2(t)$: $|F_{2,i}(t)|\le N_{2,i}\,\forall i=\overline{1,n}$ is a smooth enough vector-disturbance with the positive constants $N_1,N_{2,i}$.

We wish to specify the sliding mode controller

$$u=\begin{cases}u^+,\Im(x,e,t)>0,\\ u^-,\Im(x,e,t)<0,\end{cases} \tag{32}$$

where

$$\Im(x,e,t)=0 \tag{33}$$

is an equation of sliding surface as a dynamic operator acting on state variable vector x and output tracking error $e(t)=y^*(t)-y(t)$; u^+,u^- - continuous functions of x,t, in order to accomplish the following goal:

- the output $y(t)$ of the nonlinear nonminimum phase system, Eq. (31), must asymptotically track the given reference output trajectory $y^*(t)$ in a sliding mode. That is $\lim_{t\to\infty}\|y^*(t)-y(t)\|=\lim_{t\to\infty}\|e(t)\|=0$, and the output tracking must be linear with given eigenvalues placement in the dynamic sliding surface from Eq. (33).

Utilizing a nonsingular transformation[6,7]

$$\begin{bmatrix}x^1\\ x^2\end{bmatrix}=\begin{bmatrix}M^1\\ M^2\end{bmatrix}x,\ M=\begin{bmatrix}M^1\\ M^2\end{bmatrix},\ MB=\begin{bmatrix}0\\ b_2\end{bmatrix},\ MAM^1=\begin{bmatrix}A^{11}A^{12}\\ A^{21}A^{22}\end{bmatrix},$$

$$b_2\neq 0,\ MF=\begin{bmatrix}f_1(t)\\ \Delta A(x^1,x^2)+f_2(t)\end{bmatrix},\ GM^{-1}=\begin{bmatrix}G^1 \vdots G^2\end{bmatrix}$$

of the system in Eq. (31) to a regular form[6,7], the system, Eq. (31), is rewritten in the following format:

$$\begin{cases}\dot{x}^1=A_{11}x^1+A_{12}x^2+f_1(t)\\ \dot{x}^2=A_{21}x^1+A_{22}x^2+\Delta A(x^1,x^2)+f_2(t)+b_2u\\ y=G_1x^1+G_2x^2,\end{cases} \tag{34}$$

where $x^1\in R^{n-1},x^2\in R^1,b_2\neq 0$. Considering a state variable x^2 as a virtual control, the transfer function

$$\frac{Y(s)}{X_2(s)}=G_1[sI-A_{11}]^{-1}A_{12}+G_2 \tag{35}$$

is assumed to have zeros in the right hand half of the complex plane. It is obvious that a direct application of a conventional sliding mode controller to an output tracking in the system (34) is not possible because of the nonminimum phase nature of the plant[6,7,13]

Considering a state variable x^2 as a virtual control input at the first step of the sliding mode controller design, the dynamic sliding manifold is introduced as follows:

$$\Im = x^2 + \sigma = 0\,. \tag{36}$$

The function σ is designed as a dynamic operator, acting on the output tracking error e :

$$\sigma = W(s)e = \frac{Q(s)}{P(s)}e\,, \tag{37}$$

where $Q(s)$ and $P(s)$ are polynomials of the Laplace variable s.

Assuming that the sliding mode exists in the system, Eq. (34), on the dynamic sliding surface, Eq. (36), the motion of the system in Eq. (34) in this surface is described as follows[6,7]:

$$\dot{x}^1 = A_{11}x^1 - A_{12}\sigma + f_1(t), \quad y = G_1x^1 - G_2\sigma \tag{38}$$

The evolution of $e(t)$ in the system, Eq. (38), was identified in operator notation. This is

$$\begin{aligned}&\left[P(s) - G_2Q(s) - G_1\left(sI - A_{11}\right)^{-1}A_{12}Q(s)\right]E(s) = \\ &= P(s)Y^*(s) - G_1\left(sI - A_{11}\right)^{-1}P(s)F_1(s).\end{aligned} \tag{39}$$

The polynomials $P(s)$ and $Q(s)$ are designed in order to

- provide any desired roots' placement to the characteristic equation

  $$P(s) - G_2Q(s) - G_1\left(sI - A_{11}\right)^{-1}A_{12}Q(s) = 0\,, \tag{40}$$

- reject the effect of the reference profile $y^*(t)$ and the unmatched disturbance $f_1(t)$ to the output tracking error e in a steady state.

The rejection condition is derived as follows:

$$e_{ss} = \lim_{s\to 0} s\frac{P(s)Y^*(s) - G_1\left(sI - A_{11}\right)^{-1}P(s)F_1(s)}{P(s) - G_2Q(s) - G_1\left(sI - A_{11}\right)^{-1}A_{12}Q(s)} = 0\,. \tag{41}$$

A control function u is designed to meet the well known[6,7] sliding mode existing condition

$$\Im\cdot\dot{\Im} < -\rho|\Im|, \quad \rho > 0 \tag{42}$$

Eq. (42) is calculated for the system in Eq. (30). This is

$$\begin{cases} u^+ < -\dfrac{1}{b_2}\left[\dot{\sigma} + \rho + A_{21}x^1 + A_{22}x^2 + \Delta A(x^1,x^2) + f_2(t)\right] \\ u^- > -\dfrac{1}{b_2}\left[\dot{\sigma} - \rho + A_{21}x^1 + A_{22}x^2 + \Delta A(x^1,x^2) + f_2(t)\right] \end{cases} \tag{43}$$

where b_2 is assumed to be positive. To make Eq. (43) realizable, we need to make $\dot{\sigma}$ bounded. Obviously

$$\dot{\sigma} = L^{-1}\left\{s\frac{Q(s)\left[P(s)Y^*(s) - G_1\left(sI - A_{11}\right)^{-1}P(s)F_1(s)\right]}{P(s)\left[P(s) - G_2Q(s) - G_1\left(sI - A_{11}\right)^{-1}A_{12}Q(s)\right]}\right\} \tag{44}$$

is bounded if

- the output tracking profile $y^*(t)$ and the disturbance $f_1(t)$ are bounded,
- the denominator of Eq. (44) does not have roots in the right half of the complex plane, and has at most one root in the origin.

Consequently, the sliding mode exists in the nonminimum phase system (31) via SMC Eqs. (32) and (43) in the dynamic sliding manifold

$$\Im = M^2x + \frac{Q(s)}{P(s)}e = 0\,, \tag{45}$$

SMC Design

The mathematical model, Eq. (30), was considered for a generic strategic air defense missile interceptor at Mach 4.5 and altitude 10 *km*. This is

$$\begin{cases} \dot{\theta} = q \\ \dot{q} = -387.3\theta + 387.4\gamma - 2.59q - 4382\delta + f_2(t) \\ \dot{\gamma} = 1.239\theta - 1.239\gamma - 0.0068\cos\gamma + 0.471\delta + f_1(t) \\ y = \gamma \end{cases}. \tag{46}$$

This system is transformed to a regular form of Eq. (34) as follows:

$$\begin{cases} \dot{z} = -0.823z + 0.823\theta - 0.0019q - \\ \quad -0.0068\cos(z - 0.001075q) + f_1(t) - 0.001075f_2(t) \\ \dot{\theta} = q \\ \dot{q} = 387.3z = 387.4\theta - 0.675q - 4382\delta + f_2(t) \\ y = z - 0.001075q \end{cases} \tag{47}$$

The transfer function Eq. (35) identified for the system in Eq. (47) is

$$\frac{Y(s)}{q(s)} = -0.001075\frac{(s - 26.5)(s + 28.9)}{s(s + 0.823}\,, \tag{48}$$

which has a zero in the right hand side of the complex plane and, therefore, is nonminimum phase. The dynamic sliding surface, Eq. (36), is designed as follows:

$$\Im = q + \sigma = 0\,. \tag{49}$$

The system's Eq. (47) motion in the sliding surface of Eq. (49) is identified in accordance with Eq. (38). This is

$$\begin{cases} \dot{z} = -0.823z + 0.823\theta + 0.0019\sigma + \psi(z,\sigma,t) \\ \dot{\theta} = -\sigma \\ y = z - 0.001075q \end{cases} \tag{50}$$

where $\psi(z,q,t) = -0.0068\cos\gamma + f_1(t) - 0.001075f_2(t)$.

Assuming $y^*(t) = y_{1d}$ and $\psi(z,q,t)$ are piece-wise constants, the following polynomials $P(s)$ and $Q(s)$ were identified

$$\begin{cases} Q(s) = -287.19s^2 - 2259.36s - 8399.39 \\ P(s) = s(s+16.91) \end{cases}. \tag{51}$$

It is easy to show that the identified $P(s)$ and $Q(s)$ meet Eq. (41), and make the characteristic equation of the system Eq. (50)

$$\begin{aligned} &[P(s) + 0.001975Q(s)]s^2 + \\ &+[0.823P(s) + 0.00275Q(s)] - 0.823Q(s) = 0 \end{aligned} \tag{52}$$

to satisfy ITAE criterion

$$s^4 + 2.1\omega s^3 + 3.4\omega^2 s^2 + 2.7\omega^3 s + \omega^4 = 0 \tag{53}$$

with $\omega = 10\, rad/s$. So, the following dynamic sliding surface is designed

$$\Im = q - \frac{287.19s^2 + 2259.36s + 8399.39}{s(s+16.91)} e. \tag{54}$$

The control law for Eqs. (32) and (43) is designed in a simplified format. This is

$$\delta = \delta_{\max} sign \Im, \tag{55}$$

where

$$\begin{cases} \delta_{\max} > |\delta_{eq}| + \rho, \quad \rho > 0, \\ \delta_{eq} = \dot{\sigma} + 387.3z - 387.3\theta - 0.675q. \end{cases} \tag{56}$$

Eliminating control chattering, the discontinuous control law in Eq. (55) is substituted by

$$\delta = \delta_{\max} sat \frac{\Im}{\varepsilon}, \quad \varepsilon > 0. \tag{57}$$

Simulation

The cruise missile type interceptor Eq. (46) was simulated with the dynamic SMC Eqs. (54) and (57) and $\delta_{\max} = 0.532\, rad$ and $\varepsilon = 1$. A low pass filter with a transfer function $\frac{1}{1+0.03s}$ filters the dynamic sliding surface. The results are shown in figures 12 through 14. It is observed that the tracking of the sensor with noise line-of-sight angle profile by the flight path angle is insensitive to the disturbances $f_1(t)$ and $f_2(t)$.

Conclusion

We have shown the development of a high performance hit-to-kill interceptor control system, addressing hostile environment, which includes noises and disturbances, based on sliding mode control techniques. We have addressed the control of both ballistic and cruise missile type interceptors. After the interceptor flight path angle has captured the line-of-sight angle, the SMC will guide the interceptor along the line-of-site angle until the target missile is impacted. The SMC control algorithms we have developed do not require estimations of the target position, velocity and acceleration. The result of this development is robust, relatively simple controllers for TMD missile applications. Future work needs to be accomplished to develop high fidelity SMC algorithms for incorporation into TMD missile simulations and systems.

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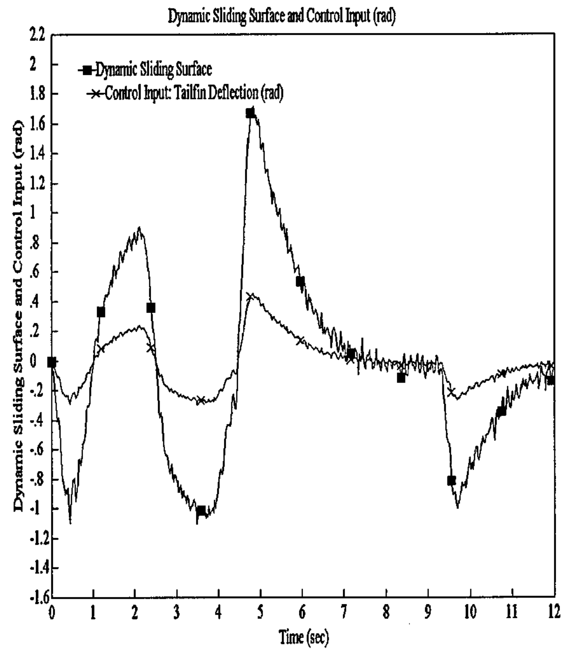

Fig. 13 Control input: tailfin deflection with disturbances and noise of measurement (cruise missile type interceptor)

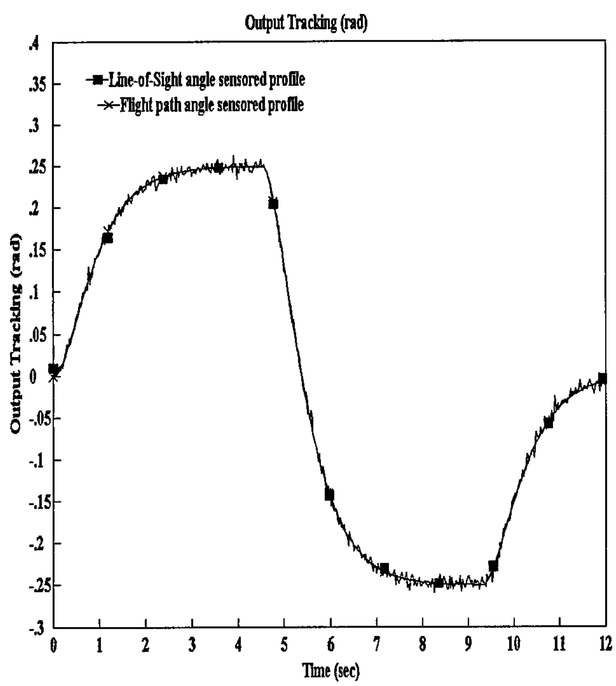

Fig. 12 Tracking of line-of-sight angle with disturbances and noise of measurement (cruise missile type interceptor)

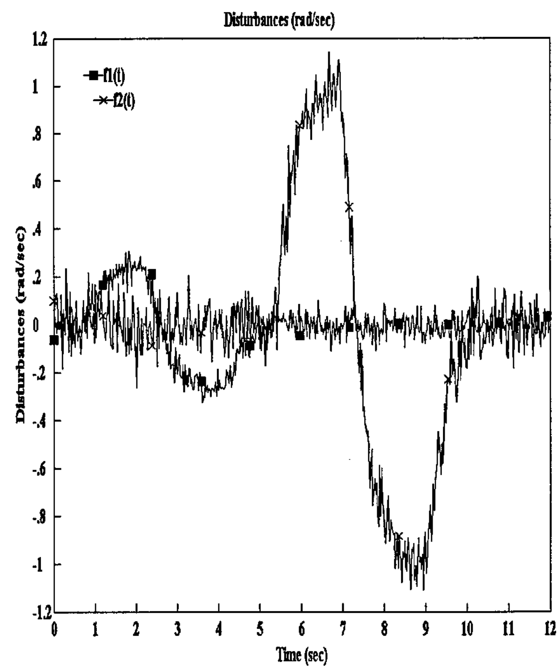

Fig. 14 Disturbances (cruise missile type interceptor)